Bounds on $\sin 2\beta$ and $|V_{ub}/V_{cb}|$ from the Light-Quark Triangle

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Abstract

With the help of the light-quark triangle, which is essentially congruent to the rescaled unitarity triangle for a variety of textures of the quark mass matrices, we calculate the CP-violating quantity $\sin 2\beta$ and the ratio of $|V_{ub}|$ to $|V_{cb}|$. We find that $\sin 2\beta$ is most likely to lie in the range $0.45 \leq \sin 2\beta \leq 0.60$, a result compatible very well with the present BaBar and Belle measurements. On the other hand, $|V_{ub}/V_{cb}| \geq 0.8$ is disfavored. Our bounds on both $\sin 2\beta$ and $|V_{ub}/V_{cb}|$ can soon be confronted with more precise data to be accumulated from the asymmetric B-meson factories.

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Recently the BaBar and Belle Collaborations have updated their measurements of $\sin 2\beta$, where $\beta \equiv \arg[-(V_{cb}^* V_{cd})/(V_{tb}^* V_{td})]$ is an inner angle of the unitarity triangle of quark flavor mixing [1], from the CP-violating asymmetry in B_d^0 vs $\bar{B}_d^0 \to J/\psi K_S$ decays:

$$\sin 2\beta = \begin{cases} 0.34 \pm 0.20(\text{stat}) \pm 0.05(\text{syst}) \,, & (\text{BaBar [2]}) \,, \\ \\ 0.58^{+0.32}_{-0.34}(\text{stat})^{+0.09}_{-0.10}(\text{syst}) \,, & (\text{Belle [3]}) \,. \end{cases} \tag{1}$$

These results are lower than, but not in conflict with the previous result reported by the CDF Collaboration: $\sin 2\beta = 0.79 \pm 0.42$ [4]; and they are also compatible with the results obtained from global analyses of the unitarity triangle in the standard model [5]. Although the central value from the BaBar measurement is relatively lower than those from the Belle and CDF measurements, there is no serious discrepancy that one can really claim. In comparison with the preliminary data announced last year by the BaBar and Belle Collaborations [6], their present data have considerably narrowed the possible room for new physics to manifest itself in the CP-violating asymmetry between $B_d^0 \to J/\psi K_S$ and $\bar{B}_d^0 \to J/\psi K_S$ decays [7]. A new window is on the other hand being opened, with the help of more precise data on $\sin 2\beta$ and other CP-violating parameters to be accumulated at the B-meson factories, towards stringent tests of the texture of quark mass matrices. Reliably quantitative information on quark mass matrices will shed light on the underlying flavor symmetry and its breaking mechanism, which are crucial for our deeper understanding of the origin of quark masses, flavor mixing, and CP violation.

The main purpose of this paper is to determine $\sin 2\beta$ from the so-called light-quark triangle, whose shape depends only upon the flavor mixing between (u, c) and (d, s) quarks in the heavy quark limit [8]. As shown in Ref. [9], the light-quark triangle is essentially congruent to the rescaled unitarity triangle for a variety of realistic quark mass matrices. Therefore it is possible to calculate the angles of the unitarity triangle, which are observable parameters of CP violation, from the sides of the light-quark triangle. We find that the numerical prediction of $\sin 2\beta$ from the light-quark triangle is very well consistent with the present BaBar and Belle data. We also obtain a very instructive bound on $|V_{ub}/V_{cb}|$, although it is somehow lower than the currently most favorable experimental value. Our results of $\sin 2\beta$ and $|V_{ub}/V_{cb}|$ can soon be confronted with more precise data to be accumulated at the KEK and SLAC B-meson factories.

Let us start with a brief retrospection of the light-quark triangle derived from the quark mass matrices with specific texture zeros. In the standard model or its extensions which have no flavor-changing right-handed currents, one can always choose a specific flavor basis in which both the up-type quark mass matrix M_u and its down-type counterpart M_d are Hermitian and have vanishing (1,3) and (3,1) elements [8]. Such a flavor basis is quite natural in the sense that it coincides with the observed hierarchy of quark masses. Without loss of generality, the (1,1) element of M_u or M_d can also be arranged to vanish through a proper but physically irrelevant transformation of the chosen flavor basis [10]. It is impossible, however, to arrange the (1,1), (1,3) and (3,1) elements of both M_u and M_d to vanish in the most general case. Hence quark mass matrices of the form

$$M_q = \begin{pmatrix} 0 & D_q & 0 \\ D_q^* & C_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix} \,, \tag{2}$$

where q = u (up-type) or d (down-type), keep the essential generality except for assuming the simultaneous vanishing of the (1,1) elements in M_u and M_d. In view of the strong mass
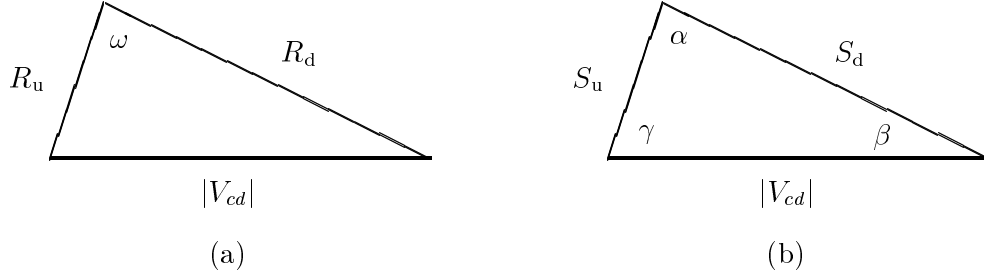
Figure 1: In the complex plane: (a) the light-quark triangle (LT); and (b) the rescaled unitarity triangle (UT).

hierarchy in each quark sector, one naturally expects that $|A_q|$ is dominant over $|B_q|$, $|C_q|$ and $|D_q|$ in magnitude. It turns out that the heavy quark limit (i.e., $m_t \to \infty$ and $m_b \to \infty$), which allows the light quarks (u, c) or (d, s) to be decoupled from the massive t or b quark, is a useful and realistic approximation. Then the flavor mixing matrix element $|V_{us}|$ or $|V_{cd}|$ can be derived from the mismatch between the diagonalization of M_u and that of M_d:

$$|V_{cd}| = |R_u - R_d \exp(i\omega)| , \tag{3}$$

where

$$R_u = \sqrt{\frac{m_u}{m_u + m_c}} \sqrt{\frac{m_s}{m_d + m_s}} ,$$

$$R_d = \sqrt{\frac{m_c}{m_u + m_c}} \sqrt{\frac{m_d}{m_d + m_s}} , \tag{4}$$

and $\omega \equiv \arg(D_d) - \arg(D_u)$. Such an instructive relation was discussed long time ago [11] to interpret the Cabibbo mixing between (u, c) and (d, s) quarks. Note that Eq. (3) defines a triangle in the complex plane, the so-called light-quark triangle as illustrated in Fig. 1(a). This triangle has been shown to be approximately congruent to the rescaled unitarity triangle in Fig. 1(b) [8, 9], defined by the relation

$$|V_{cd}| = |S_u - S_d \exp(i\alpha)| , \tag{5}$$

where $S_u = |V_{ub}^* V_{ud}/V_{cb}^*|$, $S_d = |V_{tb}^* V_{td}/V_{cb}^*|$, and $\alpha \equiv \arg[-(V_{tb}^* V_{td})/(V_{ub}^* V_{ud})]$ is another inner angle of the unitarity triangle. As a result, the phase parameter ω, which is only relevant to the magnitude of flavor mixing between (u, c) and (d, s) quarks in the heavy quark limit, may lead to CP violation (i.e., $\omega \approx \alpha$) once the heavy quark limit is slightly lifted. One can then make use of the light-quark triangle to calculate the angles of the unitarity triangle in a good approximation. Since the former only involves $|V_{cd}|$, m_u/m_c and m_d/m_s, it is possible to predict the value of $\sin 2\beta$ with rather small numerical uncertainties.

A particularly interesting case is $\omega = 90°$; i.e., the light-quark triangle is a right-angled triangle [12, 13]. In this special case, we can estimate the magnitude of $\sin 2\beta_{LT}$ by use of a rather simple relation:

$$\tan \beta_{LT} = \frac{R_u}{R_d} \approx \sqrt{\frac{m_u m_s}{m_c m_d}} . \tag{6}$$

Taking $m_u/m_c = 4 \cdot 10^{-3}$ and $m_d/m_s = 0.05$ typically [1], we obtain $\beta_{LT} \approx 15.8°$ or $\sin 2\beta_{LT} \approx 0.52$. The latter is fairly consistent with experimental data given in Eq. (1).
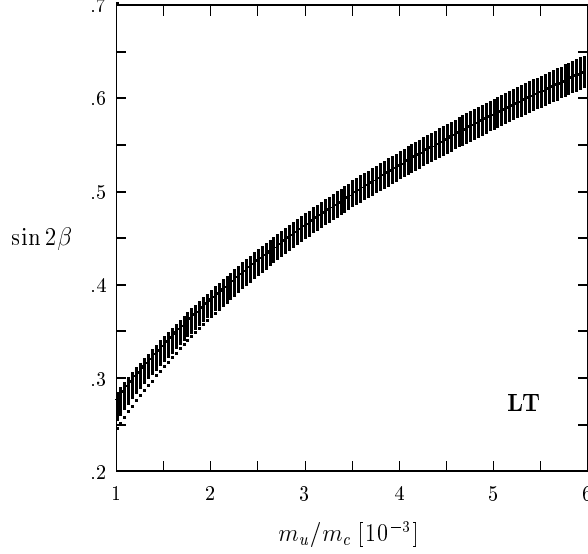
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Figure 2: Bound on $\sin 2\beta$ from the light-quark triangle (LT), where $|V_{cd}| = 0.222 \pm 0.009$ and $m_s/m_d = 18.9 \pm 0.8$ have been input. The solid curve corresponds to the central values of the input parameters.

More generally, one may calculate the CP-violating angle β from the light-quark triangle with the help of the cosine theorem. We obtain

$$\cos \beta_{\mathrm{LT}} = \frac{1}{2} \sqrt{\frac{m_s}{m_d}} \left[|V_{cd}| + \frac{1}{|V_{cd}|} \left(\frac{m_d}{m_s} - \frac{m_u}{m_c} \right) + \Delta_{\mathrm{LT}} \right] \tag{7}$$

in the next-to-leading order approximation, where

$$\Delta_{\mathrm{LT}} = \frac{1}{2} \left[|V_{cd}| - \frac{1}{|V_{cd}|} \left(\frac{m_d}{m_s} - \frac{m_u}{m_c} \right) \right] \left(\frac{m_u}{m_c} + \frac{m_d}{m_s} \right) . \tag{8}$$

Then a numerical prediction for $\sin 2\beta_{\mathrm{LT}}$ as a function of m_u/m_c can be made by taking $|V_{cd}| = 0.222 \pm 0.009$ [2] and $m_s/m_d = 18.9 \pm 0.8$ [15]. Note that the ratio m_s/m_d, unlike the ratio m_u/m_c, can be determined rather accurately using the chiral perturbation theory. We plot the result in Fig. 2. We see that the magnitude of $\sin 2\beta_{\mathrm{LT}}$ increases monotonically with the value of m_u/m_c. Corresponding to the generous range of m_u/m_c (i.e., $1 \cdot 10^{-3} \leq m_u/m_c \leq 6 \cdot 10^{-3}$), $\sin 2\beta_{\mathrm{LT}}$ takes values from 0.25 to 0.65. The uncertainties resulting from the errors of m_s/m_d and $|V_{cd}|$ are insignificant, as illustrated in Fig. 2. It is obvious that the bound on $\sin 2\beta$ from the light-quark triangle is very well compatible with the present BaBar and Belle data.

Note that $\sin 2\beta \geq 0.50$ is expected to hold in the standard model with current data [16]. This lower bound implies $m_u/m_c \geq 3.5 \cdot 10^{-3}$. Indeed the most probable values of m_u/m_c lie in the range $3 \cdot 10^{-3} \leq m_u/m_c \leq 5 \cdot 10^{-3}$ [1]. Accordingly we arrive at a rather narrow range for $\sin 2\beta$ from the light-quark triangle: $0.45 \leq \sin 2\beta_{\mathrm{LT}} \leq 0.6$. Such an interesting range

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[2]Note that we have adopted the average of the experimental values $|V_{cd}| = 0.224 \pm 0.016$ and $|V_{us}| = 0.2196 \pm 0.0023$ [1] as the input of $|V_{cd}|$. The reasons are simply that (a) $|V_{cd}| = |V_{us}|$ holds in the heavy quark limit; and (b) $|V_{us}| - |V_{cd}| \sim \mathcal{O}(10^{-4})$ holds in reality, as guaranteed by the unitarity of the quark flavor mixing matrix [14].
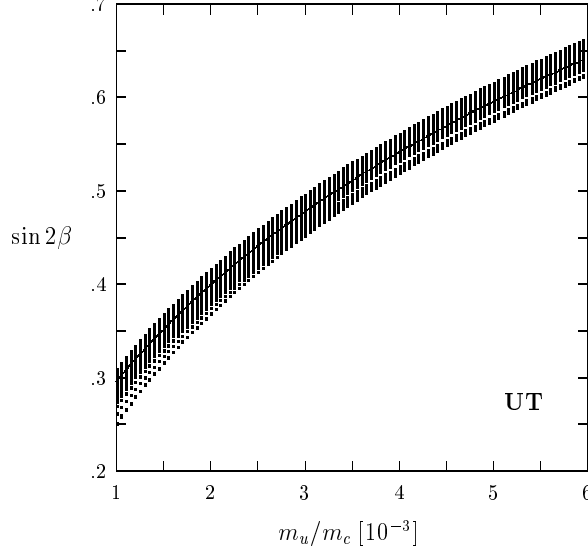
Figure 3: Bound on $\sin 2\beta$ from the rescaled unitarity triangle (UT), where $|V_{cd}| = 0.222 \pm 0.009$, $m_s/m_d = 18.9 \pm 0.8$ and $m_b/m_s = 34 \pm 4$ have been input. The solid curve corresponds to the central values of the input parameters.

can be further narrowed, once our knowledge on the mass ratio m_u/m_c is improved [17]. After more precise data are accumulated from the B-meson factories at KEK and SLAC, it will be possible to make a stringent test of $\sin 2\beta$ derived from the light-quark triangle.

Now we discuss the possibility to determine $\sin 2\beta$ directly from the unitarity triangle, based on the quark mass matrices in Eq. (2). The specific deviation of the rescaled unitarity triangle in Fig. 1(b) from the light-quark triangle in Fig. 1(a) cannot be calculated, unless a further assumption is made for the texture of M_u and M_d. In a number of phenomenological models with natural flavor symmetries [18], $|B_q| \sim |C_q|$ and $|B_u|/C_u = |B_d|/C_d$ have been assumed. In this case, we obtain

$$
\begin{aligned}
S_u &= \sqrt{\frac{m_u}{m_c}} \left[1 - \frac{1}{2}|V_{cd}|^2 + \frac{1}{2}\frac{m_c m_s}{m_u m_b}\left(\frac{m_u}{m_c} + \frac{m_d}{m_s} - |V_{cd}|^2\right)\right] , \\
S_d &= \sqrt{\frac{m_d}{m_s}} \left(1 + \frac{1}{2}\frac{m_u}{m_c} - \frac{1}{2}\frac{m_d}{m_s}\right)
\end{aligned}
\tag{9}
$$

in the next-to-leading order approximation [9]. We see that $S_u \approx R_u$ and $S_d \approx R_d$ hold to the leading-order degree of accuracy [12]. Applying the cosine theorem to Fig. 1(b) leads to

$$
\cos\beta_{\mathrm{UT}} = \frac{1}{2}\sqrt{\frac{m_s}{m_d}} \left[|V_{cd}| + \frac{1}{|V_{cd}|}\left(\frac{m_d}{m_s} - \frac{m_u}{m_c}\right) + \Delta_{\mathrm{UT}}\right]
\tag{10}
$$

in the same order approximation, where

$$
\Delta_{\mathrm{UT}} = \Delta_{\mathrm{LT}} - \frac{1}{2|V_{cd}|}\sqrt{\frac{m_s}{m_d}}\left(\frac{m_u}{m_c} + \frac{m_d}{m_s} - |V_{cd}|^2\right)\frac{m_s}{m_b} .
\tag{11}
$$

It is clear that $\Delta_{\mathrm{UT}} = \Delta_{\mathrm{LT}}$ appears in the limit $m_b \to \infty$. With the inputs given above as well as $m_b/m_s = 34 \pm 4$ [19], one can similarly compute $\sin 2\beta_{\mathrm{UT}}$ as a function of m_u/m_c. The
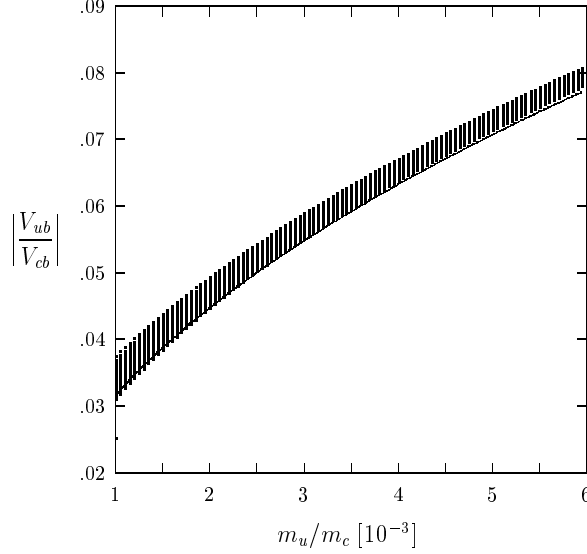
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Figure 4: Bound on $|V_{ub}/V_{cb}|$ from the light-quark triangle (the solid curve) and the rescaled unitarity triangle (the shaded region), where $|V_{cd}| = 0.222 \pm 0.009$, $m_s/m_d = 18.9 \pm 0.8$ and $m_b/m_s = 34 \pm 4$ have been input.

numerical result is illustrated in Fig. 3. We find that the magnitude of $\sin 2\beta_{UT}$ is slightly larger than that of $\sin 2\beta_{LT}$, as a consequence of the new correction term induced by the mass ratio m_s/m_b in Δ_{UT}. Taking $m_u/m_c = 4 \cdot 10^{-3}$ for example, we obtain $0.52 \leq \sin 2\beta_{UT} \leq 0.56$ in contrast with $0.51 \leq \sin 2\beta_{LT} \leq 0.54$. This confirms that one may use the sides of the light-quark triangle to calculate the angles of the unitarity triangle to a good degree of accuracy, without involving further details of the quark mass matrices.

Note that an experimental value of $\sin 2\beta$ lower than that obtained from the global fit of the unitarity triangle may have important implications on some parameters of the standard model [16, 20, 21]. In particular, the true value of $|V_{ub}/V_{cb}|$ might be somehow smaller than the presently most favorable value (i.e., $|V_{ub}/V_{cb}| \approx 0.09$ [1]). It is therefore desirable, in the near future at B-meson factories, to check the self consistency between the experimental measurements of $\sin 2\beta$ and $|V_{ub}/V_{cb}|$ within the framework of the standard model. Given the texture of quark mass matrices in Eq. (2), the magnitude of $|V_{ub}/V_{cb}|$ can be calculated from either the light-quark triangle [3] or the unitarity triangle. We obtain

$$
\begin{aligned}
\left|\frac{V_{ub}}{V_{cb}}\right|_{LT} &= \sqrt{\frac{m_u}{m_c}} \ , \\
\left|\frac{V_{ub}}{V_{cb}}\right|_{UT} &= \sqrt{\frac{m_u}{m_c}} \left[1 + \frac{1}{2} \frac{m_c m_s}{m_u m_b} \left(\frac{m_u}{m_c} + \frac{m_d}{m_s} - |V_{cd}|^2 \right) \right]
\end{aligned}
\tag{12}
$$

in the next-to-leading order approximation. Clearly $|V_{ub}/V_{cb}|_{UT}$ is a little larger than $|V_{ub}/V_{cb}|_{LT}$, due to the correction from m_s/m_b. One may easily check, with the help of Eqs. (9) and (12), that $|V_{ub}/V_{cb}|_{UT} = S_u/|V_{ud}|$ holds to the same degree of accuracy. The numerical results of $|V_{ub}/V_{cb}|_{LT}$ and $|V_{ub}/V_{cb}|_{UT}$ are shown in Fig. 4. We observe that the

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[3] Note that a calculation of $|V_{ub}/V_{cb}|$ from the light-quark triangle makes sense only when the heavy quark limit is slightly lifted. In this case, $|V_{ub}/V_{cb}|_{LT}$ should be understood as the leading-order approximation of $|V_{ub}/V_{cb}|_{UT}$ determined from the unitarity triangle.

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possibility of $|V_{ub}/V_{cb}| \geq 0.08$ is strongly disfavored. For m_u/m_c changing from $3 \cdot 10^{-3}$ to $5 \cdot 10^{-3}$, we get $0.055 \leq |V_{ub}/V_{cb}|_{\text{UT}} \leq 0.074$. The experimental data obtained in the LEP experiments indicate that $|V_{ub}/V_{cb}|$ should be larger than 0.08 [22]. In our approach this can hardly be accommodated. We conclude that the LEP results for $|V_{ub}/V_{cb}|$ should be questioned. The issue will soon be clarified by the new experimental data to be obtained from the B-meson factories.

In summary, we have calculated $\sin 2\beta$ and $|V_{ub}/V_{cb}|$ from the light-quark triangle based on a generic texture of quark mass matrices. The results turn out to be good approximations of those obtained directly from the unitarity triangle. We find that the present BaBar and Belle data on $\sin 2\beta$ can well be interpreted in the context of our model of quark masses and CP violation. More accurate numerical predictions have to rely on further progress in determining the mass ratio m_u/m_c. A crucial experimental test of the texture of quark mass matrices under discussion will be the improved measurement of $|V_{ub}/V_{cb}|$. Our bounds on both $\sin 2\beta$ and $|V_{ub}/V_{cb}|$ can soon be confronted with more precise data to be accumulated from the asymmetric B-meson factories at KEK and SLAC.

References

[1] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[2] BaBar Collaboration, B. Aubert *et al.*, hep-ph/0102030 (submitted to Phys. Rev. Lett. for publication).

[3] Belle Collaboration, A. Abashian *et al.*, hep-ph/0102018 (submitted to Phys. Rev. Lett. for publication).

[4] T. Affolder *et al.* (CDF Collaboration), Phys. Rev. D **61**, 072005 (2000).

[5] S. Plaszczynski and M.H. Schune, hep-ph/9911280; A. Ali and D. London, hep-ph/0002176; F. Caravaglios, F. Parodi, P. Roudeau, and A. Stocchi, hep-ph/0002171; and references therein.

[6] BaBar Collaboration, B. Aubert *et al.*, hep-ex/0008060; Belle Collaboration (H. Aihara), hep-ex/0010008.

[7] See, e.g., A.L. Kagan and M. Neubert, Phys. Lett. B **492**, 115 (2000); J.P. Silva and L. Wolfenstein, hep-ph/0008004; G. Eyal, Y. Nir, and G. Perez, JHEP **0008**, 028 (2000); Z.Z. Xing, hep-ph/0008018; A.J. Buras and R. Buras, hep-ph/0008273; A. Masiero, M. Piai, and O. Vives, hep-ph/0012096; R. Fleischer and T. Mannel, hep-ph/0101276.

[8] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **413**, 396 (1997); Phys. Rev. D **57**, 594 (1998).

[9] H. Fritzsch and Z.Z. Xing, Nucl. Phys. B **556**, 49 (1999);

[10] G.C. Branco, D. Emmanuel-Costa, and R.G. Felipe, Phys. Lett. B **477**, 147 (2000).

[11] H. Fritzsch, Phys. Lett. B **70**, 436 (1977); **73**, 317 (1978); S. Weinberg, in *Transaction of the New York Academy of Sciences*, **38**, 185 (1977); F. Wilczek and A. Zee, Phys. Lett. B **70**, 418 (1977).

[12] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **353**, 114 (1995); and references therein.

[13] A recent study of this interesting possibility and its implication on $\sin 2\beta$ can also be found in: M. Randhawa and M. Gupta, hep-ph/0011388 (to appear in Phys. Rev. D).

[14] Z.Z. Xing, Phys. Rev. D **51**, 3958 (1995); J. Phys. G. **23**, 1563 (1997).

[15] H. Leutwyler, Phys. Lett. B **378**, 313 (1996).

[16] A.J. Buras, hep-ph/0101336; A.J. Buras and R. Buras, in Ref. [7].

[17] R. Barbieri, L.J. Hall, and A. Romanino, Nucl. Phys. B **551**, 93 (1999).

[18] For a recent review with extensive references, see: H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000); hep-ph/9912358.

[19] S. Narison, Phys. Lett. B **358**, 113 (1995); hep-ph/9911454; and private communications.

[20] Y. Nir, hep-ph/0008226; G. Eyal, Y. Nir, and G. Perez, in Ref. [7].

[21] J. Ellis. hep-ph/0011396.

[22] LEP Working Group on $|V_{ub}|$, http://battagl.home.cern.ch/battagl/vub/vub.html.